

# THE THERMODYNAMIC RELATIONSHIP BETWEEN SURFACE TEMPERATURE AND WATER VAPOR CONCENTRATION IN THE TROPOSPHERE

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## **ABSTRACT**

The theoretical and empirical thermodynamics discussed in this paper explain the physics behind the observed reduction in relative humidity in the upper troposphere as surface temperature and surface humidity increase. This contradicts the physics embedded in current GCM models commonly used by the climate science community. The key to the physics discussed in this paper is the understanding of the relationship between water vapor condensation and the resulting PV work energy distribution under the influence of a gravitational field. New analyses of empirical, observational radiosonde data are presented which show the relationship between thermal energy and PV work energy resulting from this water vapor condensation process.

## **INTRODUCTION**

There has been much discussion in the literature concerning the role of water vapor in the thermodynamics of the atmosphere. In the past two decades or so, this discussion has focused on the role of water vapor as a positive feedback for the radiative forcing supposedly caused by increasing CO<sub>2</sub> in the atmosphere. A key assumption in this argument is that the relative humidity in the atmosphere will remain constant as the atmosphere heats or cools [1, 2]. In the case of atmospheric heating, this means that the specific humidity (g water vapor/kg air) or mixing ratio (g water vapor/kg dry air) will increase as the surface/atmosphere warms. This is based on the Clausius-Clapeyron equation which defines the increase in water evaporation as surface temperature increases. This is one of the algorithms that is included in all Global Climate Models (GCM) currently in use. There are various model and empirical estimates that range from 3% to 7% of increased evaporation for every degree Celsius increase in surface temperature [1, 2, 8].

Unfortunately the actual observational data contradict this core rationale of AGW. While specific humidity levels in the lower troposphere do increase with increasing surface temperature, the specific humidity levels in the mid to upper troposphere have shown a decreasing trend over the past few decades even though surface temperatures (and tropospheric temperatures) are thought to be increasing. This is illustrated in Figures 1 and 2.

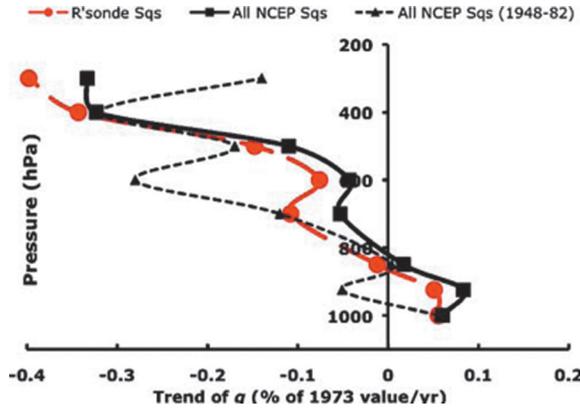


Figure 1. Comparison of the profile of the specific humidity trends obtained as an average of all National Centers for Environmental Prediction (NCEP) tropical grid squares and the profile obtained as an average only of those tropical squares in which there is an input of actual radiosonde data for the full 35-year period. Also shown is the profile obtained as an average of all tropical NCEP grid squares but for the period 1948 to 1982 (Partridge, et al. Theor Appl Climatol, February 2009).

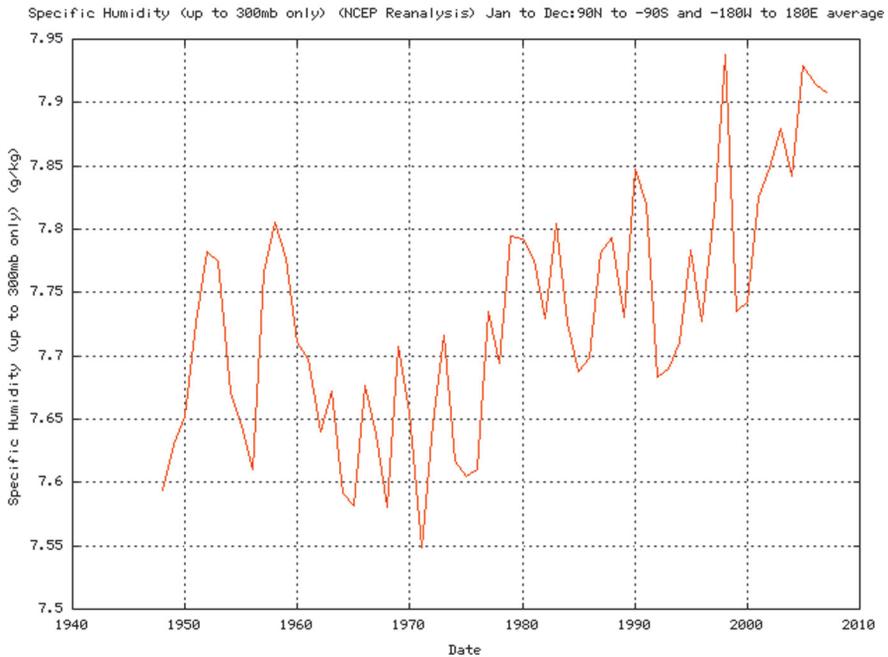


Figure 2a. 1000 mb (surface).

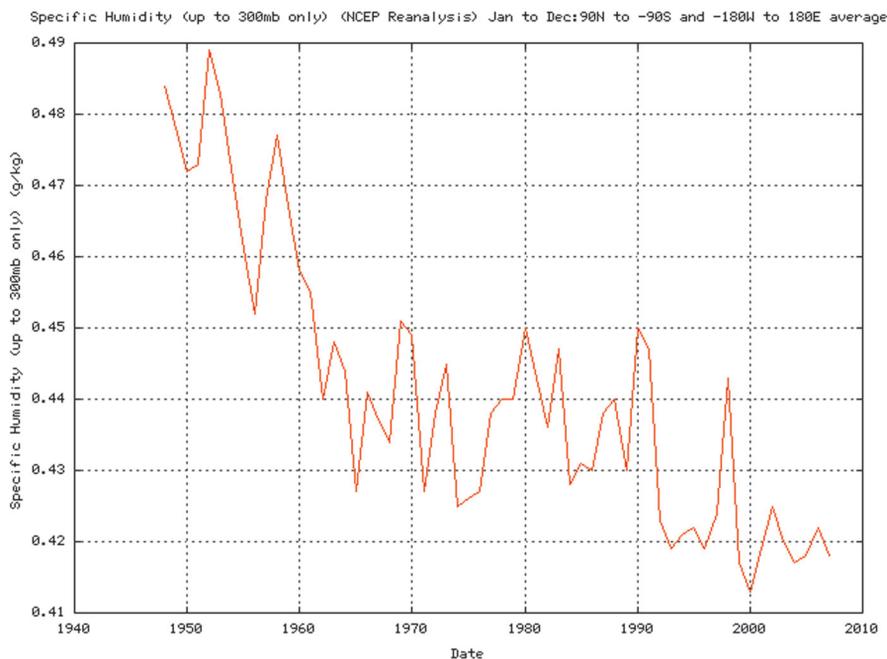


Figure 2b. 400 mb (7 Km).

Figures 2a. and 2b. From NOAA Earth System Research Laboratory database (ESRL) for surface and 7 km elevations. (Note: the reference to 300mb at the top of each chart refers to the maximum elevation available for the ESRL humidity measurement database)

As stated in the abstract of Partridge, et al, 2009 [3]: “The upper-level negative trends in  $q$  are inconsistent with climate-model calculations..... Negative trends in  $q$  as found in the NCEP data would imply that long-term water vapor feedback is negative—that it would reduce rather than amplify the response of the climate system to external forcing such as that from increasing atmospheric  $CO_2$ ”. Thus, there is something amiss in the physics embedded in the climate models. Solomon et al., 2010 [4] have also noted a drop in water vapor concentrations in the stratosphere over the last decade. This may or may not be related to the water vapor decline in the troposphere outlined by Partridge.

The rest of this paper will 1) outline the pertinent classical thermodynamics/meteorology and 2) present new, empirical, observational radiosonde data analyses that explain this humidity profile phenomenon.

### THE PHYSICS BEHIND THE HUMIDITY PROFILE IN THE TROPOSPHERE

The physics/thermodynamics that describe the empirical findings in this paper have long existed in the classical texts of thermodynamics/meteorology. But it should be worthwhile to review them prior to looking at the observational radiosonde data analysis that explains the humidity profiles we see in the troposphere.

First let's start with the basic process equation for the first law of thermodynamics (Note that all units of measure for energy in this discussion assume intensive properties, i.e., per unit mass):

$$dU = dQ - PdV \quad (1)$$

where  $dU$  is the change in total internal energy of the system,  $dQ$  is the change in thermal energy of the system and  $PdV$  is work done to or by the system on the surroundings.

But equation (1) applies to a system that is not influenced by external fields. Since the atmosphere is under the influence of a gravitational field the first law equation must be modified to account for the potential energy portion of internal energy that is due to position:

$$dU = dQ + gdh - PdV \quad (2)$$

where  $g$  is the acceleration of gravity ( $9.8 \text{ m/s}^2$ ) and  $h$  is the mass particle vertical elevation relative to the earth's surface.

The thermal energy component of the system ( $dQ$ ) can be broken down into two distinct parts: 1) the molecular thermal energy due to its kinetic/rotational/ vibrational internal energies ( $CvdT$ ) and 2) the intermolecular thermal energy resulting from the phase change (condensation/evaporation) of water vapor ( $Ldq$ ). Thus the first law can be rewritten as:

$$dU = CvdT + Ldq + gdh - PdV \quad (3)$$

where  $C_v$  is the specific heat capacity at constant volume,  $L$  is the latent heat of condensation/evaporation of water ( $2257 \text{ J/g}$ ) and  $q$  is the mass of water vapor available to undergo the phase change.

The relationship between  $T$ ,  $P$  and  $V$  is also defined in the ideal gas law which is applicable in the predominately diatomic gaseous composition of the troposphere:

$$PV = nRT \quad (4)$$

Since the troposphere has been repeatedly shown in various meteorological texts to very closely approximate adiabatic behavior, we can, for the purposes of this discussion, equate  $dU = 0$  in equation (3). Thus while any of the various components of internal energy in any given air parcel can vary, any variation in any given component must be offset by a corresponding inverse change in other components. The total internal energy will remain constant. As we shall see,  $PV$  work energy is very important to the understanding of this thermodynamic behavior of the atmosphere, and the thermodynamic role of water vapor condensation plays an important part in this overall energy balance. But this is unfortunately often overlooked or ignored in the more recent climate science literature. The atmosphere is a very dynamic system and cannot be adequately analyzed using static, steady state mental models that primarily focus only on thermal energy.

If we begin with an air parcel at the surface at a given surface temperature, there is an equilibrium relation between liquid water at the surface and water vapor concentration in the air parcel. The water vapor concentration ( $q$ ) will depend on the

surface temperature, the air temperature and the air pressure. The value of  $q$  is established at this point.

If the surface temperature is greater than the air temperature, the surface air parcel will increase in temperature. As the air temperature increases the air parcel will respond according to equation (4) and  $PV$  will also increase. But since the atmosphere is not physically constrained, the system behaves as a constant pressure system and the increase in  $PV$  will all be primarily embodied as  $\Delta V$ . The increase in  $V$  will immediately result in buoyancy of the air parcel and it will begin to rise. At this point the air parcel is for all practical purposes in adiabatic isolation (no exchange of energy with its surroundings except for  $PV$  work) and equation (3) is now operable (remember that  $dU = 0$ ). Due to the existence of the gravitational field, there is a natural, vertical, decreasing pressure gradient from the surface to the tropopause. As a result,  $dV$  will increase as the parcel rises (decreasing  $P$ ) resulting in  $PV$  work on its surroundings. But since energy is conserved in equation (3), any increase in  $PdV$  (a negative term when work is done on the surroundings) will result in a decrease in parcel temperature (thermal energy,  $CvdT$ , is expended to provide the  $PV$  work) and a corresponding increase in potential energy ( $gdh$ ) due to the parcel vertical displacement ( $PV$  work is exchanged for potential energy). This exchange of internal energy components (thermal energy to  $PV$  work energy to potential energy) will continue as long as the parcel has positive buoyancy and rises. Thermal energy ( $CvdT$ ) will decrease and potential energy ( $gdh$ ) will increase.

We have now accounted for 3 of the 4 independent variables in equation (3) in describing the process known as convection. Heat has been transferred from the surface to the mass particles in the air parcel and the heated air/mass parcel is being transported vertically. Convection is both a heat and mass transfer process. Once heat is transferred, the driving force for the mass transfer is the gravity induced decreasing  $\Delta P$  profile with altitude. (Note: the larger scale meridional distribution (advection, zonal temperature gradient, Equator-to-poles heat transfer etc.) is not being covered in this analysis but these can also be explained via classical thermodynamics/meteorological treatments).

Now we can deal with the latent heat component of equation (3). The baseline value of  $q$  (specific humidity) is established at the surface where the parcel may or not be in steady state equilibrium with the surface liquid and saturated. If saturated, some of the water vapor will almost immediately begin to condense during convection due to the decreasing temperature (offset somewhat by the decreasing pressure). If not saturated, condensation will begin at a higher altitude. Condensation releases a very large quantity of heat as compared to other heat transfer processes. Compare the value of latent heat, 2257 J/g, to the heat capacity of water vapor, 2.1 J/g-K. Condensation can alter the convection process (heat and mass transfer) by orders of magnitude. When condensation occurs, heat is immediately transferred to the air parcel mass, dramatically increasing  $T$ .  $Ldq$  decreases and  $CvdT$  increases. But as we saw at the surface, any increase in  $T$  must result in a corresponding increase in  $PV$  (equation (4)). Thus the thermal energy transferred from  $Ldq$  to  $CvdT$  must also result in a transfer of thermal energy from  $CvdT$  to work energy  $PdV$ . This whole process will result in an increase in temperature but possibly an even larger increase in  $PV$  work. Thus

understanding this PV/thermal energy ratio profile is critical in understanding the thermodynamics of the troposphere.

This ratio of thermal energy released versus PV work energy created is the crux of the physics behind the troposphere humidity trend profile versus surface temperature. But what is it that controls this energy ratio? It turns out that the same factor that controls the pressure profile in the troposphere also controls the tropospheric temperature profile and the PV/thermal energy ratio profile. That factor is gravity. If you take equation (3) and modify it to remove the latent heat term, and assume for an adiabatic, ideal gas system  $C_p T = C_v T + PV$ , you can easily derive what is known in the various meteorological texts as the “dry adiabatic lapse rate”:

$$dT/dh = -g/C_p = 9.8 \text{ K/km} \quad (5)$$

Thus the temperature profile of a steady state dry troposphere is solely a function of the heat capacity of the troposphere and the gravitational constant. This profile is independent of the temperature of the troposphere itself. This is also the equilibrium lapse rate for a steady state dynamic flow where no latent heat is involved. Thus the atmosphere tries to reach a dynamic equilibrium or an equilibrium rate of change [5, 9]. This lapse rate is established by the physics surrounding the convective flow process that was described at the beginning of this section. As an air parcel expands and rises, thermal energy ( $C_v dT$ ) is converted to PV work energy ( $PdV$ ) which results in an increase in potential energy ( $gdh$ ). Thermal energy is traded off for potential energy (conservation of energy). Without the internal input of latent heat the temperature declines at a rate of 9.8 K/km. Take a thermal snapshot of the dry ascending parcel every km and you will see this temperature profile or very close to it.

Keeping this gravitational steady state equilibrium in mind, let’s look again at what happens when latent heat is released (condensation) during air parcel ascension. Latent heat release immediately increases the parcel temperature. But that also results in rapid PV expansion which then results in a drop in parcel temperature. Buoyancy results and the parcel ascends and is driven by the descending pressure profile created by gravity. The rate of ascension, and the parcel temperature, is a function of the quantity of latent heat released and the PV work needed to overcome the gravitational field to reach a dynamic equilibrium. The more latent heat that is released, the more rapid the expansion/ascension. And the more rapid the ascension, the more rapid is the adiabatic cooling of the parcel. Thus the PV/thermal energy ratio should be a function of the amount of latent heat available for phase conversion at any given altitude. The corresponding physics shows the system will try to force the convecting parcel to approach the dry adiabatic or “gravitational” lapse rate as internal latent heat is released.

For the water vapor remaining uncondensed in the parcel, saturation and subsequent condensation will occur at a more rapid rate if more latent heat is released. In fact if the cooling rate is sufficiently large, super saturation can occur, which can then cause very sudden condensation in greater quantity. Thus the thermal/PV energy ratio is critical in determining the rate of condensation occurring. The higher this ratio, the more complete is the condensation in the parcel, and the lower the specific humidity will be at higher elevations.

### Empirical Observations to Verify the Physics

Figures 3, 4 and 5 represent an analysis of three sets of radiosonde soundings taken at Miami, Florida in 2009. I downloaded this data last year as part of another project. (Radiosonde sounding raw data are courtesy of the University of Wyoming, College of Engineering, Department of Atmospheric Science). The soundings were taken on the morning and evening of 6/18/09 and on the morning of 6/27/09 approximately one month later. The soundings represent three varying levels of both surface temperature and specific humidity (mixing ratio). As surface temperature increases so does the specific humidity, as would be expected from the Clausius-Clapeyron equation.

The bold solid line curve in each graph is the actual temperatures recorded during the radiosonde sounding. The dotted line curve (Dry Adiat) represents the theoretical

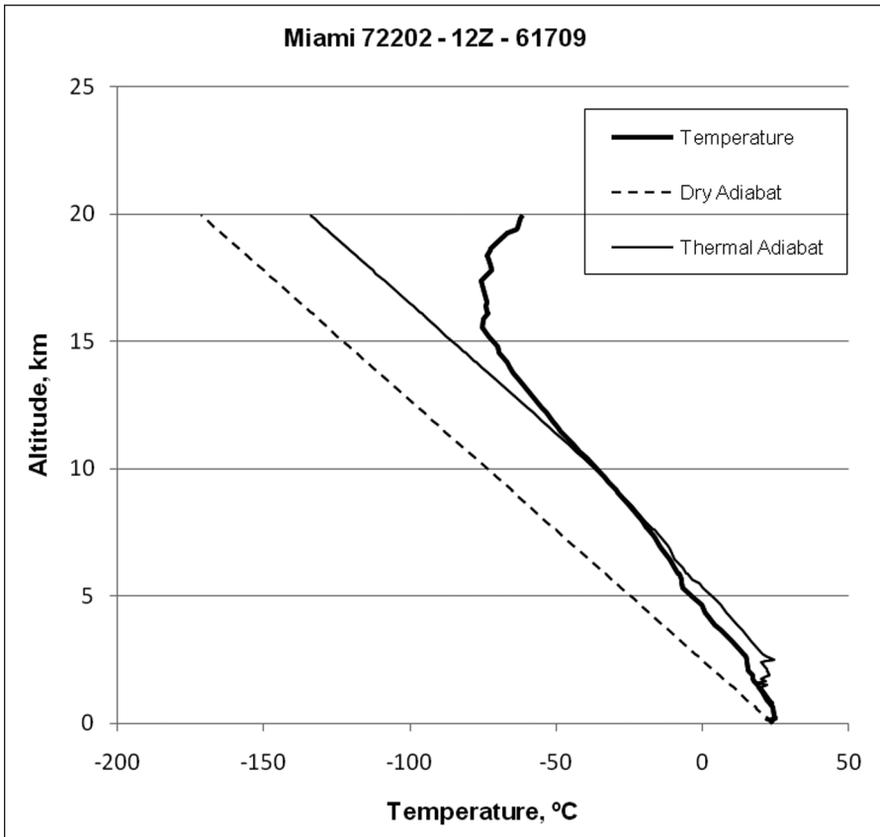


Figure 3. Miami 72202 – 12Z – 61709

- Date/Time: 6/18/09 7AM
- Surface Temperature (1000 hPa) = 23.6 °C
- Surface Mixing Ratio (g water vapor/kg dry air at 1000 hPa) = 17.07
- Lower Troposphere Lapse Rate (°C/km) = 7.0

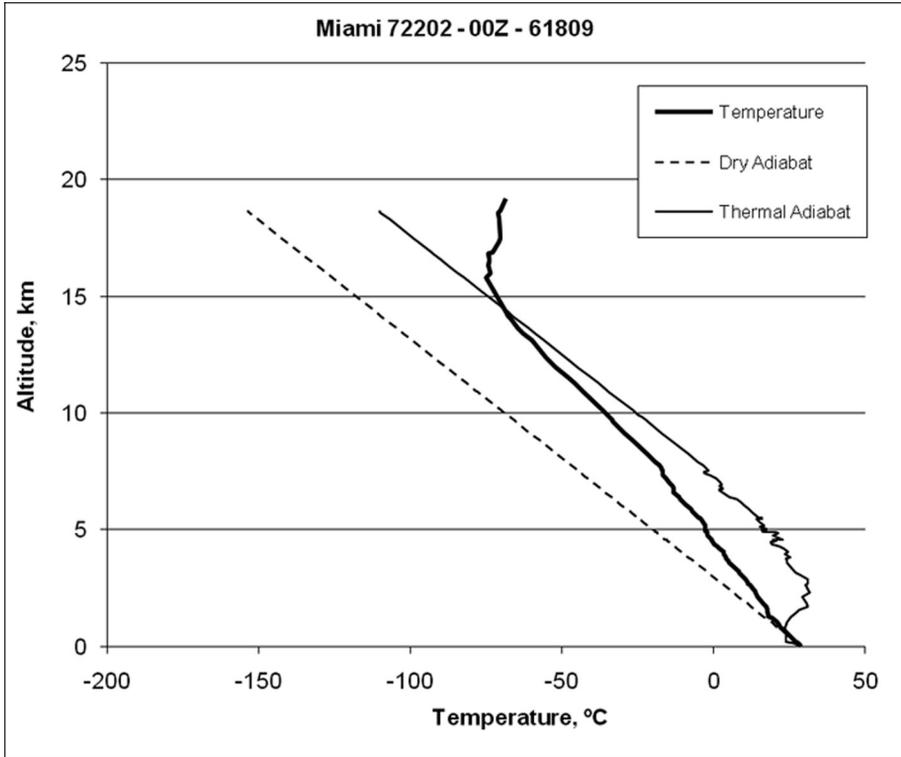


Figure 4. Miami 72202 – 00Z – 61809

- Date/Time: 6/18/09 7PM
- Surface Temperature (1000 hPa) = 27.8 °C
- Surface Mixing Ratio (g water vapor/kg dry air at 1000 hPa) = 20.06
- Lower Troposphere Lapse Rate (°C/km) = 5.9

dry adiabatic lapse rate of 9.8 °C/km. The thin solid line curve (Thermal Adabat) is the temperature profile that would occur if all latent heat released at all altitudes were converted 100% to thermal energy (temperature). This is calculated based on the reduction in mixing ratio at each reading at altitude and is added to the temperature of the Dry Adabat at that altitude. This is cumulative beginning at the surface. Thus the  $\Delta T$  between the Dry Adabat and the actual temperature represents the latent heat that was released and converted to thermal energy. The  $\Delta T$  between the actual temperature and the Thermal Adabat represents the latent heat that was released and converted to PV energy. The Dry Adabat is the baseline since it is the theoretical temperature profile if  $Ldq = 0$ .

There are several general observations that should be noted:

- The surface temperature differential between the maximum and minimum soundings is 4.2 °C with the 7PM temperature being the highest.

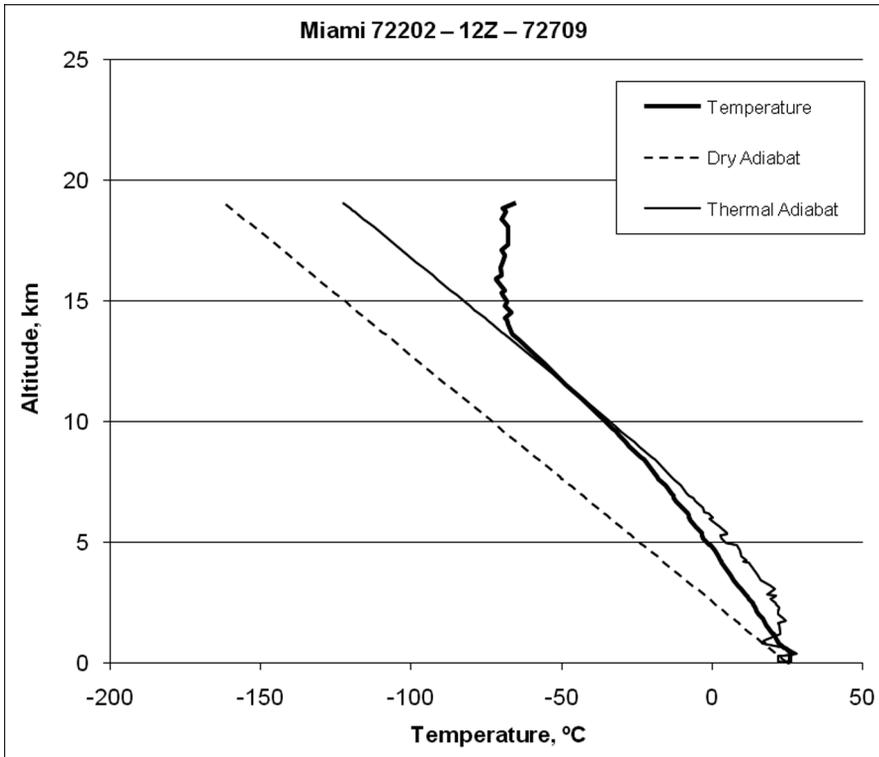


Figure 5. Miami 72202 - 12Z - 72709

- Date/Time: 7/27/09 7AM
- Surface Temperature (1000 hPa) = 25.8 °C
- Surface Mixing Ratio (g water vapor/kg dry air at 1000 hPa) = 18.17
- Lower Troposphere Lapse Rate (°C/km) = 6.3

- The surface mixing ratio differential between the maximum and minimum soundings is 2.99 g/kg with 7PM being the highest. This roughly represents ~4.2% increase in specific humidity for every degree increase in temperature, but this varies between the soundings.
- The lower troposphere lapse rate decreases (slower rate of cooling) with increasing system surface humidity levels, as expected. But the differences in lapse rate are far less than expected based on the relative release of latent heat occurring in the three systems.
- The amount of PV work that occurs during ascension increases markedly as the system surface humidity levels increase, especially at lower altitudes. This is quantified below.
- PV work energy (and thus latent heat release) is at a maximum between 2.0 km and 2.5 km.

- Latent heat release is effectively complete at 7.5 km for the highest surface humidity system (20.06 g/kg) but continues up to 11 km for the lower surface humidity systems (18.17 and 17.07 g/kg). The higher humidity system has seen complete condensation at a lower altitude, and a significantly higher temperature ( $-17\text{ }^{\circ}\text{C}$ ) than the lower humidity systems ( $\sim -40\text{ }^{\circ}\text{C}$ ) despite the much greater quantity of latent heat released.

This leads to the following conclusions:

- Due to the existence of the gravitational field and the resulting pressure profile, the system will always try to approach a dynamic equilibrium represented by the dry adiabatic lapse rate (black curve).
- When a higher concentration of surface water vapor exists, which subsequently condenses thus releasing more latent heat, more work energy ( $\mathbf{PdV}$ ) will be needed to convectively cool the parcel and move the temperature profile closer to the dry adiabatic lapse rate.
- The wetter system will convert a greater portion of the released latent heat towards work energy ( $\mathbf{PdV}$ ) versus thermal energy ( $\mathbf{CvdT}$ ). The *rate* of expansion/convective flow will increase and the *rate* of condensation will increase.
- Note that the wet adiabatic lapse rate does not reach the dry adiabatic lapse rate even after condensation ceases. This is partly, if not primarily, due to subsiding air that is converting potential energy to thermal energy via adiabatic heating which is mixing with the upward convecting air. The subsiding air is increasing temperature at  $+9.8\text{ }^{\circ}\text{C}/\text{km}$ , the dry adiabatic lapse rate).

Now, if we look more closely at the distribution of latent heat conversion to thermal and PV energy, the difference between the three soundings becomes striking (Figure 6). For the lower surface humidity systems (7AM), the percentage of latent heat energy that is converted to thermal energy is always greater than that which is converted to PV work energy. The latent heat conversion to PV energy reaches a maximum at  $\sim 40\%$ . The maximum PV/Thermal energy ratio is 0.8 for these two systems. However in the higher surface humidity system (7PM) the latent heat conversion is overwhelmingly directed to PV work energy at the lower elevations, exceeding 80% at its maximum. The maximum PV/Thermal energy ratio is 4.3, more than five times that of the lower surface humidity systems. This results in a marked increase in the rates of expansion, convection and adiabatic cooling and thus a marked increase in the rate of condensation for the higher surface humidity system.

If we look at the cumulative latent heat release for the three systems, the thermodynamic behavioral differences become clearer (Figure 7). The higher the surface humidity in a system, the more latent heat it has to release and it releases it at a much faster rate. The 7PM weather system (20.06 g/kg) has released as much latent heat at approximately 5 km as the lower humidity systems have released at 7 km and 10 km respectively. This reflects the much larger PV/thermal energy ratio and suggests a much more intense convective expansion and velocity. Condensation occurs much more rapidly and efficiently as surface humidity of the system increases.

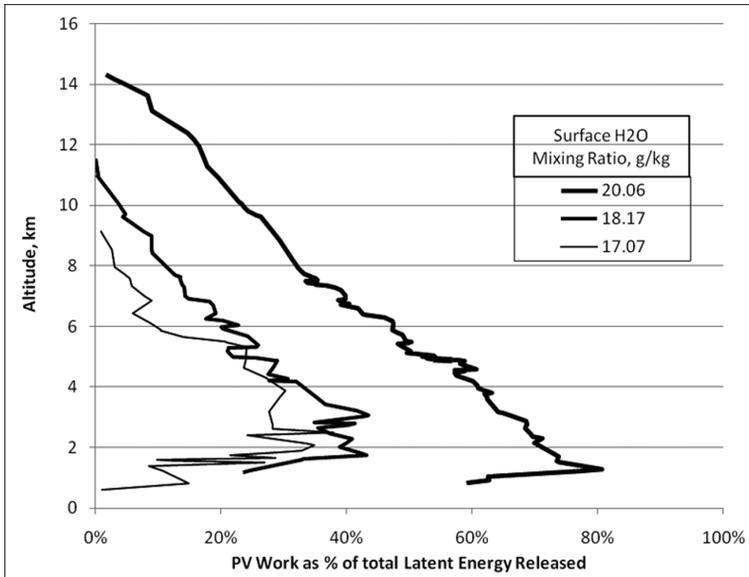


Figure 6. PV Work Energy as a Percentage of Latent Heat Energy Released.

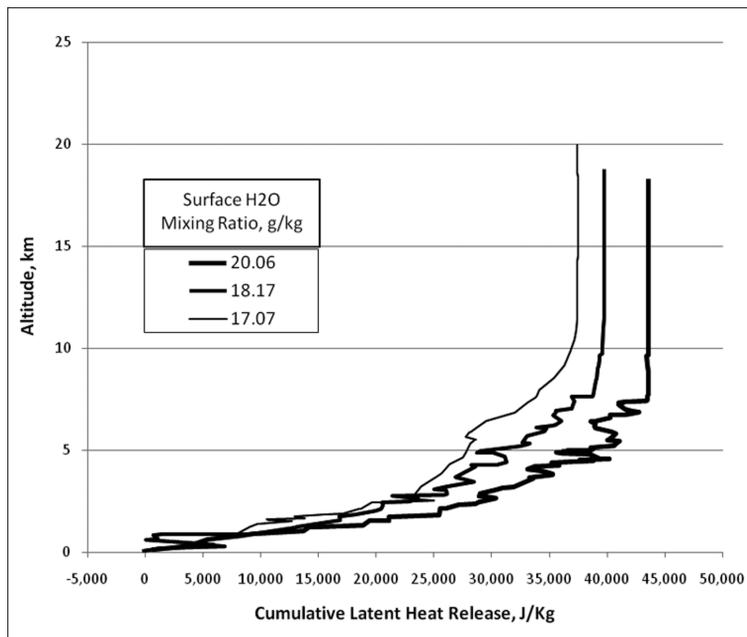


Figure 7. Cumulative Latent Heat Release.

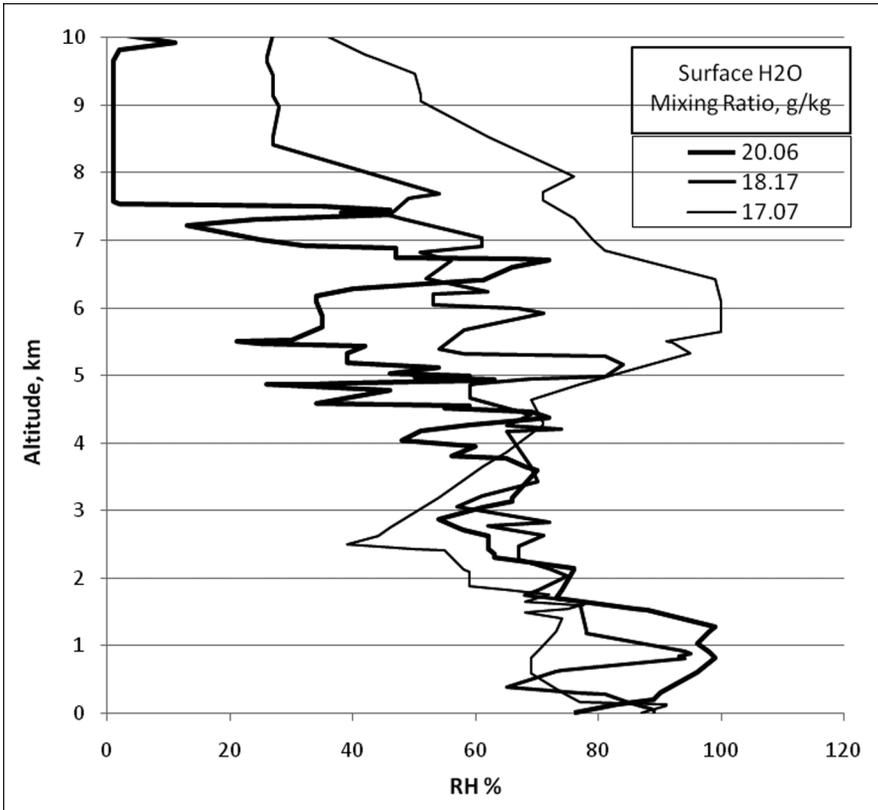


Figure 8. Relative Humidity vs. Altitude.

But how does all this translate into humidity profiles at different altitudes? This is illustrated in Figure 8. At lower altitudes (below 4 km), all three systems show a gradual but erratic decline in RH with increasing altitude. But at higher altitudes (above 4 km) the three systems begin to diverge with the higher surface humidity systems showing a higher rate of RH decrease. In contrast the lowest surface humidity system (17.07 g/kg) shows an actual increase in RH at middle troposphere elevations. At higher elevations, the differences in RH between the three systems become more and more pronounced. The wettest system at the surface actually approaches zero RH in the upper troposphere.

## CONCLUSION

The theoretical and empirical physics/thermodynamics outlined in this paper predict that systems having higher surface temperatures will show higher humidity levels at lower elevations but lower humidity levels at higher elevations. This is demonstrated in the empirical decadal observational data outlined in the Introduction, in the daily radiosonde data analysis discussed above and explained by classical thermodynamics/meteorology relationships. This theoretical and empirical analysis contradicts the current GCM

computer models. The physics embedded in the GCM models predict a constant relative humidity throughout the troposphere as surface temperature increases. As a result, these models imply a positive feedback from water vapor as surface temperature and specific humidity increase. The key to understanding the actual observed atmospheric humidity profile is to properly take into account the physics/thermodynamics of PV work energy in the atmosphere resulting from the release of latent heat of condensation under the influence of the gravitational field. The current physics underlying the GCM models appear to be incorrect. If anything, the actual behavior of water vapor with increasing surface temperature as outlined in this paper would imply a negative feedback. The observational data leaves little room for doubt

The contents of this paper may also have relevance for the work of Ferenc Miskolczi [6, 7] in that his radiosonde analysis shows that over the decades, as CO<sub>2</sub> concentrations have increased, water vapor concentrations at higher elevations have decreased yielding offsetting IR absorbance properties for the two greenhouse gases. That offset results in a constant Optical Depth in the troposphere. The results in this paper, while supporting the decadal decreasing water vapor concentration in the upper atmosphere, apply to any increase in surface temperature for whatever cause. It is not specific to CO<sub>2</sub> forcing. The same physics/thermodynamic processes will dominate whether the increasing surface temperature is due to CO<sub>2</sub>, solar variability, PDO, AMO, cosmic rays, etc...

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